61223

Second Semester B.A./B.Sc. Degree Examinations, September/October 2021

(CBCS – Freshers and Repeaters)

Paper II – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer ALL Parts.

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) On the set Z, $*$ is defined by $a*b=a+b+2$, $\forall a,b\in z$, find the identity element.

(b) Prove that in a group G, $(a^{-1})^{-1}=a$, $\forall a\in G$.

(c) Find the angle between the radius vector and the tangent to the curve $r=a\,e^{\theta\cot\alpha}$.

(d) Find the polar subtangent for the curve $r=a\sec 2\theta$.

(e) Find the asymptotes parallel to coordinate axes to the curve $x^2y^2-a^2x^2=a^2y^2$.

(f) Find $\dfrac{ds}{dx}$ for the curve $y^2=4ax$.

(g) Verify the exactness of the equation $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$.

(h) Find the general solution of the equation $y=3px+p^2y^2$.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) If Q^+ is the set of all positive rationals, prove that $(Q^+,*)$ is an Abelian group where $*$ is defined by $a*b=\dfrac{2ab}{3}$, $\forall a,b\in Q^+$.

(b) Show that set of all cube roots of unity forms an Abelian group with respect to multiplication

(c) Prove that a non-empty subset H of a group $(G,*)$ is a subgroup of G if and only if $\forall a,b\in H$, $a*b^{-1}\in H$.

Or

3. (a) Prove that identity element of a group is unique.

(b) Prove that $H = \{1, 2, 4\}$ is a subgroup of the group $G = \{1,2,3,4,5,6\}$ under multiplication modulo 7.

(c) In a set $S = \{a,b,c,d\}$, if $f = \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix}$ and $g = \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix}$ then find $f \circ g$, $g \circ f$ and $f^{-1} \circ g^{-1}$.

PART – C

Answer any **TWO** full questions : **(2 × 15 = 30)**

4. (a) With usual notations, prove that $\tan\phi = r\dfrac{d\theta}{dr}$ for the polar curve $r = f(\theta)$.

(b) Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect orthogonally.

(c) Find the pedal equation of the curve $r^n = a^n \cos n\theta$.

Or

5. (a) Find the angle between the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$.

(b) Derive the formula for radius of curvature in parametric form.

(c) Find the coordinates of the centre of curvature at the point (x, y) on the curve $y^2 = 4ax$.

6. (a) Find the position and nature of the double points of the curve.

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$

(b) Find all the asymptotes of the curve

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$$

(c) Find the perimeter of the cardioide $r = a(1+\cos\theta)$.

Or

7. (a) Show that the envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$ where $ab=c^2$ is $4xy=c^2$.

(b) Find the surface area of the solid generated by revolving about the y-axis the curve $x=y^3$ from $y=0$ to $y=2$.

(c) Find the volume of the solid generated by revolving the astroide $x^{2/3}+y^{2/3}=a^{2/3}$ about the x-axis.

PART – D

Answer any **ONE** full question : **(1 × 15 = 15)**

8. (a) Solve : $\frac{dy}{dx}+\frac{2}{x}y=x^3$.

(b) Verify for exactness and solve $(2xy+3y)dx+(x^2+3x)dy=0$.

(c) Find the general and singular solution of $\sin px\cos y-\cos px\sin y=p$.

Or

9. (a) Solve : $\frac{dy}{dx}-\frac{y}{x}=y^2$.

(b) Solve : $xp^2+(y-x)p-y=0$.

(c) Find the orthogonal trajectories of the family of parabolas $y=ax^2$, where a is a parameter.