NUMERICALMETHODS-II

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Numerical solution of algebraic and Transcendental equations

Introduction:

The function $f(x)=a_0x^n+a_1x^{n-2}+\ldots\ldots.+a_{n-1}x+a_n$ where n is a positive integer and $a_0,a_1,a_2,\ldots\ldots,a_n$ are constants with $a_0 \neq 0$ is known as a polynomial of degree n.

Ex : $4x+ c = 0$ (linear equation)

$x^2 + 5x + 6 = 0$ (Quadratic equation)

$x^3+5x^2+6x+6=0$ (cubic equation)

The equation of the form $f(x)=0$ is called transcendental according as $f(x)$ it contains some other functions such as logarithmic ,exponential and trigonometric functions etc.

Ex: $x^4+\log(x-1)+e^x+\sin x=0$

The equation of the formf(x)=0 is called Algebraic according as $f(x)$ is purely a polynomial in x

Eg: $x^4+3x^2-4x+1=0$

Iteration process:

Numerical methods of finding approximate roots of the given equation is a repetatitive type of process known as iteration process.

Initial approximations:

Initial approximations to the root are often known from the physical considerations of the problem. Otherwise graphical methods are generally used obtain initial approximations to the root.

The following methods are used to finding solution of algebraic and Transcendental equations

1)Method of successive bisection
2)Method of false position
3)Newton- raphson method

1)Method of successive bisection

Formula : $x = (a+b)/2$

The method consists of locating the root of the equation $f(x)=0$ between 'a' and 'b' ($a<b$) if $f(x)$ is continuous in the interval $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs then there is a root between 'a' and 'b' ,let $f(a)$ be negative and $f(b)$ be positive. Then the first approximation to the root is $x_1=(a + b)/2$.

- If $f(x_1)=0$, x_1 is a root of $f(x)=0$ otherwise the root lies between 'a' and 'x'$_1$ or x_1 and 'b' accordingly as $f(x_1)$ is positive or negative then we bisect the interval as before and continue the process until the root is found to the desired accuracy.

- We notice that this method uses only the end points of the interval and not the values of $f(x)$ at these end points to obtain the next approximation to the root.

- The method is simple to use and the sequence of approximations always converges to the root for any f(x) which is continuous in the interval that contains the root.

- Then the approximate number of iterations required may be determined from the relation

$$\frac{b-a}{2^n} \leq \epsilon$$

Find a root of the equation $x^3 - 4x - 9 = 0$ using bisection method in four stages.

Solution: $f(x) = x^3 - 4x - 9 = 0$

Put $x = 0, 1, 2, 3, \ldots\ldots\ldots\ldots$

At $x=0$; $f(0) = 0^3 - 4(0) - 9 = -9$(-ve)

At $x=1$; $f(1) = 1^3 - 4(1) - 9 = -12$(-ve)

At $x=2$; $f(2) = 2^3 - 4(2) - 9 = -9$(-ve)

At $x=3$; $f(3) = 3^3 - 4(3) - 9 = 6$(+ve)

Since $f(2)$ is –ve and $f(3)$ is +ve ,a root lies between 2 and 3

There fore the first approximation root is

$x1 = (2+3)/2 = 2.5$

Then $f(x_1)=f(2.5)=(2.5)^3-4(2.5)-9 = -3.375$(-ve)

Therefore ,a root lies between 2.5 and 3

There fore the second approximation root is

$x_2 = (2.5+3)/2 = 2.75$

Then $f(x_2)=f(2.75)=(2.75)^3-4(2.75)-9 = 0.7969$(+ve)

Therefore ,a root lies between 2.5 and 2.75

There fore the third approximation root is

$x_3 = (2.5+2.75)/2 = 2.625$

Then $f(x_3)=f(2.625)=(2.625)^3-4(2.625)-9 = -1.4121$(-ve)

Therefore ,a root lies between 2.75 and 2.625

There fore the fourth approximation root is

$x_4 = (2.75+2.625)/2 = 2.6875$

hence the root is 2.6875 approximately