



DCMT – 301

III Semester B.Sc. Examination, February/March 2024

(NEP) (Freshers and Repeaters)

MATHEMATICS – III

Ordinary Differential Equations and Real Analysis – I

Time : 2½ Hours

Max. Marks : 60

Instruction : Answer **all** the Parts.

PART – A

I. Answer **any six** of the following : (6×2=12)

1) Show that $(x^2 - ay)dx + (y^2 - ax) dy = 0$ is exact.

2) Find the general solution of $y = px + \frac{a}{p}$.

3) Verify the condition for integrability $2yzdx + zxdy - xy(1 + z) dz = 0$.

4) Find particular integral of $(D^2 + 4D + 4) = e^{2x}$.

5) Show that the sequence $\left\{\frac{1}{n}\right\}$ is a monotonic decreasing.

6) Test the convergence of sequence $\left\{\frac{\log n}{n}\right\}$.

7) Test the nature of the series $\sum \frac{1}{n} \sin \frac{1}{n}$.

8) State Raabe's test for series of positive terms.



PART – B

II. Answer **any three** of the following : (3×4=12)

1) Verify for exactness and solve $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$.

2) Solve : $xp^2 + (y - x)p - y = 0$.

3) Solve : $y = 2px - yp^2$.

4) Find the general and singular solution of $y = px + \sin^{-1}p$.

5) Show that the family of parabola $y^2 = 4a(x + a)$ is self-orthogonal.

P.T.O.



PART - C

III. Answer **any three** of the following :

(3×4=12)

1) Solve : $y'' + 2y' + 5y = \sin 2x$.

2) Solve : $\frac{d^2y}{dx^2} + 4y = x^2$.

3) Solve : $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0$.

4) Solve : $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} + y = 0$ using the transformation $z = \tan^{-1}x$.

5) Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.



PART - D

IV. Answer **any three** of the following :

(3×4=12)

1) Prove that a monotonic decreasing sequence which is bounded below is convergent ?

2) Discuss the nature of sequence $\{n^{1/n}\}$.

3) Show that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2.

4) Prove that the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is monotonic increasing and bounded.

5) Test the convergence of the sequence 0.4, 0.44, 0.444, ...

PART - E

V. Answer **any three** of the following :

(3×4=12)

1) State and prove D'Alembert's ratio test for the series of positive terms.

2) Discuss the convergence of the series $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots$

3) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$.

4) Sum the series to infinity $\frac{1}{5} - \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} - \frac{1.4.7.10}{5.10.15.20} + \dots$

5) Sum to infinity of the series $\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \dots$