DCMT – 501

V Semester B.Sc. Examination, February/March 2024
(NEP) (Freshers)
MATHEMATICS
Mathematics – V
Real Analysis – II and Complex Analysis

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer **all** Parts.*

PART – A

Answer **any six** of the following : **(6×2=12)**

1. Define Lower Riemann sum and Upper Riemann sum.
2. Find the common refinement of $P_1 = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ and $P_2 = \left\{0, \frac{1}{2}, 1\right\}$.
3. Define Beta function.
4. Show that $\Gamma(n+1) = n\Gamma(n)$.
5. Show that $U = 2x - x^3 + 3xy^2$ is harmonic.
6. Show that f(z) = sinz is an analytic function.
7. Define Bilinear transformation.
8. State fundamental theorem of algebra.

PART – B

Answer **any three** of the following : **(3×4=12)**

1. If f(x) = 2x + 3 and $P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ is a partition of [0, 1], then find L(P, f) and U(P, f).
2. Show that the constant function f(x) = K is Riemann integrable over [a, b] and hence evaluate $\int_a^b f(x)dx$.

P.T.O.

3. If f : [a, b] $\to \mathbb{R}$ is bounded and P is a partition of [a, b] and K is a positive constant, then prove that
L(P, Kf) = KL(P, f) and U(P, Kf) = KU(P, f).

4. If f(x) and g(x) are Riemann integrable over [a, b], then prove that f(x).g(x) is also Riemann integrable over [a, b].

5. State and prove first mean value theorem of integral calculus.

PART – C

Answer **any three** of the following : **(3×4=12)**

1. Show that $\Gamma(n + 1) = n!$.

2. Evaluate $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.

3. State and prove duplication formula.

4. Evaluate $\int_0^{\pi/2} \cos^5\theta \sin^2\theta \, d\theta$.

5. Show that p. $\beta(p, q + 1) = q. \beta(p + 1, q)$.

PART – D

Answer **any three** of the following : **(3×4=12)**

1. Prove that the necessary condition for a function f(z) = u + iv to be analytic is $u_x = v_y$ and $u_y = -v_x$.

2. Show that $u = \frac{1}{2}\log(x^2 + y^2)$ and v = 2xy are harmonic, but u + iv is not analytic.

3. Find the analytic function f(z) = u + iv given $u - v = (x - y)(x^2 + 4xy + y^2)$.

4. If f(z) = u + iv is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$.

5. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and hence find its harmonic conjugate.

PART – E

Answer **any three** of the following : **(3×4=12)**

1. Evaluate $\int_{0}^{3+i} z^2 dz$ along the line by 3y = x.
2. State and prove Cauchy's integral formula.
3. Prove that the Bilinear transformation preserves the cross ratio of four points.
4. Discuss the transformation $w = e^z$.
5. State and prove Liouville's theorem.