DCMT – 502

V Semester B.Sc. Examination, February/March 2024
(NEP Scheme) (Freshers)
MATHEMATICS – VI
Vector Calculus and Group Theory

Time : 2½ Hours Max. Marks : 60

Instruction : *Answer **all** the Parts.*

PART – A

I. Answer **any six** of the following : **(6×2=12)**

1) Find $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right)$ where $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j}$.

2) Show that the vector field $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is Solenoidal.

3) State Gauss Divergence theorem.

4) If V is the volume of a region bounded by a closed surface 'S', then show that $\iint_S \vec{r}.\hat{n}.ds = 3V$.

5) Write the Euler's equation of the functional $I = \int_{x_1}^{x_2} f(x, y, y')dx$.

6) Find the extremal of the functional $I = \int_{x_1}^{x_2} \left(y^2 + x^2y'\right)dx$.

7) Define Normal subgroup.

8) Show that $f : (R, +) \to (R^+, \cdot)$ defined by $f(x) = e^x, \forall x \in R$ is a homomorphism.

PART – B

II. Answer **any three** of the following : **(3×4=12)**

1) Find the unit tangent vector drawn to the curve $x = t^2, y = 2t, z = -t^3$ at the point t = 1.

2) For any scalar field ϕ and any vector field $\vec{F}$, prove that $\text{div}(\phi\vec{F}) = \phi\,\text{div}(\vec{F}) + (\text{g rad}\,\phi)\cdot\vec{F}$.

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3) If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$, then find $\nabla\phi$ and $|\nabla\phi|$ at (1, –1, 2).

4) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1, 1, – 1) in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

5) If $\vec{F} = x^2y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$, then find Curl, Curl $\vec{F}$.

PART – C

III. Answer **any three** of the following : **(3×4=12)**

1) If $\vec{F} = (2y + 3)\,\hat{i} + xz\,\hat{j} + (yz - x)\,\hat{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve $x = 2t^2$, $y = t$, $z = t^3$ from the point (0, 0, 0) to the point (2, 1, 1).

2) Evaluate $\iint_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = 18z\,\hat{i} - 12\,\hat{j} + 3y\,\hat{k}$ and 'S' is the part of the plane $2x + 3y + 6z = 12$ in the first octant.

3) By using Green's theorem, evaluate $\oint_C (3x - y)\, dx + (2x + y)\, dy$, where 'C' is the circle $x^2 + y^2 = a^2$.

4) Evaluate by using Stoke's theorem, $\oint_C yzdx + zxdy + xydz$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

5) State and prove Greens' theorem in the plane.

PART – D

IV. Answer **any three** of the following : **(3×4=12)**

1) Find the extremal of the functional, $I = \int_{x_1}^{x_2} \left(y' + x^2y'^2\right)dx$.

2) Find the extremal of the functional, $I = \int_0^{\pi/2} \left(y^2 - y'^2 - 2y\sin x\right)dx$, under the end conditions $y(0) = 0$, $y\left(\pi/2\right) = 0$.

3) Define Geodesic. Prove that the Geodesic on a plane is a straight line.

4) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

5) Show that the extremal of the functional $\int_0^1 (y')^2 dx$ subject to the constraint $\int_0^1 y\, dx = 1$ and having $y(0) = 0$, $y(1) = 1$ is a parabolic arc.

PART – E

V. Answer **any three** of the following : **(3×4=12)**

1) Prove that the intersection of any two normal subgroups of a group is also a normal subgroup.

2) If $f : G \to G'$ be a homomorphism from the group (G, O) into the group (G′, *), then prove that (i) $f(e) = e'$ (ii) $f(a^{-1}) = [f(a)]^{-1}$, $\forall\, a \in G$. Where e and e′ are the identity elements of G and G′ respectively.

3) State and prove fundamental theorem of homomorphism.

4) If $f : G \to G'$ be a homomorphism from the group G into G′ with Kernel K. Then prove that K is a normal subgroup of G.

5) If $f = \begin{pmatrix} a & b & c & d & e \\ b & c & d & a & e \end{pmatrix}, g = \begin{pmatrix} a & b & c & d & e \\ c & a & d & e & b \end{pmatrix}$ then find $f \circ g$ and $g \circ f$.
