SDCMT201

II Semester B.Sc. Examination, June/ July - 2025
(SEP, Scheme)
MATHEMATICS
Mathematics-II

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer all Parts.

PART - A

I. Answer any Ten of the following. **(10×2=20)**

1. On Q^+, $*$ is defined by $a * b = \frac{ab}{2}$, find the identify element.

2. Define a group.

3. Find the number of generations of the cyclic group of order 24.

4. Find the right hand limit of the function $f(x) = x\sin\left(\frac{1}{x}\right)$ as x tends to 0.

5. State Cauchy's mean value theorem.

6. Evaluate $\lim_{x\to 0}\left(\frac{(1+x)^n - 1}{x}\right)$.

7. Write the relation between the cartesian and polar coordinates.

8. Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

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9. Find the asymptotes parallel to the coordinate axes for the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$.

10. Evaluate $\int_C (x+y)\,dx+(y-x)\,dy$ along the parabola $y^2 = x$ from (1, 1) to (4, 2).

11. Evaluate $\int_0^1\int_0^2 (x+y)\,dx\,dy$.

12. Evaluate $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz$.

PART - B

II. Answer any Three of the following. **(3×5=15)**

1. If Q^+ is the set of all positive rationals, prove that $(Q^+, *)$ is an abelian group, where $*$ is defined by $a*b=\frac{2ab}{3}$.

2. Prove that in a group $(G, *)$; $(a*b)^{-1}=b^{-1}*a^{-1}$, $\forall a,b\in G$.

3. Prove that in a group G, $O(a) = O(a^{-1})$, $\forall a\in G$.

4. Find all the right and left cosets of the subgroup H = {1, 3, 9} of the group $(Z_{13}-\{o\}, \otimes_{13})$.

5. State and prove Lagrange's theorem.

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PART - C

III. Answer any Three of the following. (3×5=15)

1. Prove that every function defined and continuous on a closed interval is bounded.

2. Examine the differentiability at $x = 1$ of the function $f(x)$ defined by

$$f(x)=\begin{cases} x^2, & \text{if} \quad x \le 3 \\ 6x-9, & \text{if} \quad x \ge 3 \end{cases} \text{ at } x = 3.$$

3. Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in the interval [2, 4].

4. State and prove Lagrange's mean value theorem.

5. Evaluate $\lim_{x \to 0}\left(\frac{\tan x - x}{x^2 \tan x}\right)$.

PART - D

IV. Answer any Three of the following. (3×5=15)

1. With usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$.

2. Find the Pedal equation of the curve $r^n = a^n \sin n\theta$.

3. Derive the formula for the radius of curvature in cartesian form.

4. Find the asymptotes of the curve $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$.

5. Find the position and nature of the double points of the curve $x^3 + y^3 = 3axy$.

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PART - E

V. Answer any Three of the following. (3×5=15)

1. Evaluate $\int_C (x+2y)\,dx + (4-2x)\,dy$ around the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in the counter clockwise direction.

2. Evaluate $\int_C (x+y+z)\,ds$ where C is the line joining the points (1, 2, 3) and (4, 5, 6).

3. Evaluate $\iint_R y\,dx.dy$ where R is the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

4. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration.

5. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} xyz\,dx\,dy\,dz$.