11. a) Solve by Gauss-Jacobi method,

$x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$

b) Find the largest eigen value of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ by power method.

12. a) Use Taylor's series method to find y(1.1) considering the terms up to third degree. Given that $\frac{dy}{dx} = x + y$, $y(1) = 0$.

b) Using modified Euler's method to compute y(0.1) given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ taking $h = 0.05$.

OR

13. a) Using modified Euler's method find y(0.1), given $\frac{dy}{dx} = x^2 + 1$, $y(0) = 1$.

b) Using Runge-Kutta method, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0) = 1$, compute y(0.2) taking $h = 0.2$.

VI Semester B.A./B.Sc. Examination, September 2020
(CBCS) (F + R) (2016-17 and Onwards)
MATHEMATICS (Paper – VIII)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

a) Evaluate $\lim_{z \to -2i} \dfrac{(2z+3)(z-1)}{z^2 - 2z + 4}$

b) Show that $f(z) = z^2 + 2z$ is continuous at $1 + i$.

c) Show that $u = x^3 - 3xy^2$ is harmonic.

d) Define Bilinear transformation.

e) Verify Cauchy-Reimann equations for $f(z) = \sin x \cosh y + i \cos x \sinh y$.

f) State Liouville's theorem.

g) Find the real root of the equation $x^3 - 2x - 5 = 0$ in (2, 3) with two iterations by Regula-Falsi method.

h) Write iterative formula for Runge Kutta method of fourth order.

PART – B

Answer **four full** questions. **(4×10=40)**

2. a) Find locus of the point z satisfying $\left|\dfrac{z-1}{z+i}\right| \geq 2$.

b) State and prove necessary conditions for a function $f(x, y) = u(x, y) + iv(x, y)$ to be analytic.

OR

3. a) Prove that $\lim_{z \to 0} \left(\dfrac{\bar{z}}{z}\right)$ does not exist.

b) Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$.

P.T.O.

4. a) Find the analytic function f(z) = u + iv given that $u = e^x(x\cos y - y\sin y)$.

   b) If f(z) = u + iv is an analytic function in the domain of a complex plane, then prove that $u(x, y) = C_1$ and $v(x, y) = C_2$ are orthogonal families where C_1 and C_2 are constants.

OR

5. a) If f(z) = u + iv is analytic, then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$.

   b) Show that $u = x^2 - y^2 + x + 1$ is harmonic function and find its harmonic conjugate.

6. a) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$, along $x = 2t$, $y = t^2 + 3$.

   b) If f(z) is analytic inside and on a simple closed curve 'C' and if 'a' is a point within 'C', then prove that $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$.

OR

7. a) Evaluate $\int_C \frac{1}{z(z-1)}dz$, where C is the circle |z| = 3.

   b) State and prove Cauchys integral theorem.

8. a) Show that the bilinear transformation transforms circles into circles or straight lines.

   b) Discuss the transformation w = sinz.

OR

9. a) Find the bilinear transformation which maps the points z = 1, i, –1 to w = i, 0, – i.

   b) Prove that the bilinear transformation preserves the cross ratio of four points.

PART – C

Answer **two** full questions. **(2×10=20)**

10. a) Use Bijection method in four stages to find a real root of the equation $x^3 - 2x - 5 = 0$.

    b) Using Newton-Raphson method, find the real root of $x^3 + 5x - 11 = 0$ near 1 correct to 3 decimal places.

OR