61123

First Semester B.A./B.Sc. Degree Examination, November/December 2019

(CBCS Scheme – (Freshers and Repeaters))

Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer ALL questions.

PART – A

Answer any **FIVE** questions : **(5 × 2 = 10)**

1. (a) State Cayley Hamilton Theorem.

(b) Find the value of 'λ' for which the system has a non trivial solution $2x - y + 2z = 0$, $3x + y - z = 0$, $\lambda x - 2y + z = 0$.

(c) Find the nth derivative of $\log(3x-1)$.

(d) If $z = x^3 + y^3 + 3x^2 y$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(e) Evaluate $\int_0^{\pi/2} \cos^6 x \, dx$.

(f) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$.

(g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + z + 5 = 0$.

(h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z - 16 = 0$.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ -2 & -5 & 1 & 2 \\ -3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$.

(b) Show that the equations $x+y+z=3$, $3x-5y+2z=8$, $5x-3y+4z=14$ are consistent and solve them.

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

Or

3. (a) Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ to its normal form and find the rank.

(b) For what values of λ and μ the equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(c) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

PART – C

Answer **TWO** full questions : **(2 × 15 = 30)**

4 (a) Find nth derivative of $e^{ax}\sin(bx+c)$.

(b) Find the nth derivative of $\frac{x-2}{6x^2+x-1}$.

(c) If $y=\tan^{-1}x$ then prove that $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2+n)y_n=0$.

Or

5. (a) If $u=f(r)$ where $r^2=x^2+y^2$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$.

(b) If $u=\cos^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-2\cot u$.

(c) If $u=f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

6\. (a) If $z=\sin(ax+y)+\cos(ax-y)$ prove that $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$.

(b) If $x=r\cos\theta$, $y=r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$.

(c) Evaluate $\int_0^{\pi} x\sin^4 x\cdot\cos^2 x\,dx$.

Or

7\. (a) Obtain reduction formula for $\int \tan^n x\,dx$.

(b) Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}\,dx$.

(c) Evaluate by using Leibnitz's rule of differentiation under integral sign for $\int_0^{\pi/2}\frac{dx}{\alpha(1+\cos x)}$ where α is a parameter.

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8\. (a) Find the equation of the plane bisecting the angle between the planes $7x+4y+4z+3=0$ and $2x+y+2z+2=0$.

(b) Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane containing them.

(c) Find the tangent plane to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y-z=0$.

Or

9\. (a) Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{3}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

(b) Derive the equation of right circular cone in its standard form $x^2+y^2=z^2\tan^2\alpha$.

(c) Find the equation of the right circular cylinder of radius 3 units and axis is the line $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.
