First Semester B.A./B.Sc. Degree Examination, November/December 2019

(CBCS Scheme - (Freshers and Repeaters))

Paper I - MATHEMATICS

Time: 3 Hours]

[Max. Marks: 70

Instructions to Candidates: Answer ALL questions.

PART - A

Answer any FIVE questions:

 $(5 \times 2 = 10)$

- 1. (a) State Cayley Hamilton Theorem.
 - (b) Find the value of ' λ ' for which the system has a non trivial solution 2x y + 2z = 0, 3x + y z = 0, $\lambda x 2y + z = 0$.
 - (c) Find the nth derivative of log(3x-1).
 - (d) If $z = x^3 + y^3 + 3x^2y$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
 - (e) Evaluate $\int_{0}^{\pi/2} \cos^6 x \, dx$.
 - (f) Evaluate $\int_{0}^{\pi/2} \sin^6 x \cdot \cos^4 x \, dx$.
 - (g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane x+y+z+5=0.
 - (h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 4x + 6y 8z 16 = 0$.

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PART - B

Answer **ONE** full question :

 $(1 \times 15 = 15)$

- 2. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ -2 & -5 & 1 & 2 \\ -3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$.
 - (b) Show that the equations x+y+z=3, 3x-5y+2z=8, 5x-3y+4z=14 are consistent and solve them.
 - (c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

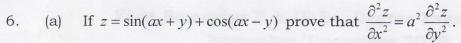
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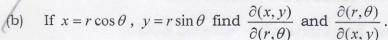
- 3. (a) Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 3 6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ to its normal form and find the rank.
 - (b) For what values of λ and μ the equations x+y+z=6, x+2y+3z=10, $x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
 - (c) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

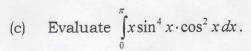
PART - C KGF - 563 122 × 15 = 30)

Answer TWO full questions:

- 4 (a) Find nth derivative of $e^{ax} \sin(bx+c)$.
 - (b) Find the nth derivative of $\frac{x-2}{6x^2+x-1}$.
 - (c) If $y = \tan^{-1} x$ then prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2+n)y_n = 0$.
- 5. (a) If u = f(r) where $r^2 = x^2 + y^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.
 - (b) If $u = \cos^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -2 \cot u$.
 - (c) If u = f(x y, y z, z x) prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.









Or
(a) Obtain reduction formula for $\int \tan^n x \, dx$.

(b) Evaluate $\int_{0}^{a} \frac{x^4}{\sqrt{a^2 - x^2}} dx$.

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(c) Evaluate by using Leibnitz's rule of differentiation under integral sign for $\int_{0}^{\pi/2} \frac{dx}{\alpha(1+\cos x)}$ where α is a parameter.

PART - D

Answer ONE full question:

 $(1 \times 15 = 15)$

8. (a) Find the equation of the plane bisecting the angle between the planes 7x+4y+4z+3=0 and 2x+y+2z+2=0.

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar and find the equation of the plane containing them.

(c) Find the tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which are parallel to the plane 2x + 2y - z = 0.

Or

9. (a) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{3} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.

(b) Derive the equation of right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.

(c) Find the equation of the right circular cylinder of radius 3 units and axis is the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.