



GN-233

103881

V Semester B.A./B.Sc. Examination, December - 2019

(CBCS) (F+R) (2016-17 and Onwards)

MATHEMATICS - VI

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** questions.

PART - A

Answer **any five** questions.

5x2=10

1. (a) Write Euler's equation when the function 'f' is independent of x and y.
- (b) Find the curve $\int_0^1 [12xy + (y')^2] dx = 0$ with $y(0) = 3$, $y(1) = 6$.
- (c) Find the function y which makes the integral $I = \int_{x_1}^{x_2} [1 + xy' + x(y')^2] dx$ an extremum.
- (d) Evaluate $\int_C xdy - ydx$, where 'C' is a line $y = x^2$ from (0, 0) to (1, 1).
- (e) Evaluate $\int_0^2 \int_0^1 (x+y) dx dy$
- (f) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$
- (g) State Gauss Divergence Theorem.
- (h) Write vector form of Green's Theorem.



P.T.O.



PART - B

Answer **two full** questions.

2x10=20

2. (a) Prove necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y') dx$, where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum that $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.

- (b) Find the extremal of the functional $I = \int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$.

OR

3. (a) Show that an extremal of $\int_{x_1}^{x_2} \left(\frac{y'}{y} \right)^2 dx$ is expressible in the form $y = ae^{bx}$.

- (b) Solve the variational problem $\delta \int_1^2 [x^2 y'^2 + 2y(x+y)] dx = 0$ with the conditions $y(1) = 0 = y(2)$.

4. (a) Find the shape of a chain which hangs under gravity between two fixed points.

- (b) Find the extremal of the functional $I = \int_0^\pi (y'^2 - y^2) dx$ under the conditions $y = 0, x = 0, x = \pi, y = 1$ subject to the condition $\int_0^\pi y dx = 1$.

OR

5. (a) Find the extremal of the functional $\int_0^1 (x + y + y'^2) dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$.

- (b) Find the geodesic on a right circular cylinder.



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PART - C



2x10=20

Answer **two full** questions.

6. (a) Evaluate $\int_C (x+2y)dx + (4-2x)dy$ along the curve $C : \frac{x^2}{16} + \frac{y^2}{9} = 1$ in anticlockwise direction.

(b) Evaluate $\iint_R xy \, dx \, dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1$.

OR

7. (a) Evaluate $\int_C (x+y+z)ds$, where 'C' is the line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x=3t+1$, $y=3t+2$, $z=3t+3$.

(b) Change the order of integration and hence evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 \, dx \, dy$.

8. (a) Find the area $\iint_S \frac{y}{x} e^x \, dx \, dy$, where S is bounded by $x=y^2$ and $y=x^2$.

(b) Find the volume of the tetrahedron by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$.

OR

9. (a) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$.

(b) If R is the region bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$. Show that $\iiint_R z \, dx \, dy \, dz = \frac{1}{24}$.

P.T.O.



PART - D

2x10=20

Answer **two full** questions.

10. (a) State and prove Green's theorem.

(b) Evaluate by Stoke's Theorem $\oint_C (yzdx + xzdy + xydz)$, where C is the curve $x^2 + y^2 = 1, z = y^2$.

OR

11. (a) Verify Green's theorem $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where 'C' is the region bounded by parabolas $y^2 = x$ and $x^2 = y$.

(b) Using divergence theorem, show that :

(i)
$$\iiint_S \vec{r} \cdot \hat{n} ds = 3V \text{ and}$$

(ii)
$$\iiint_S (\nabla r^2) \cdot \hat{n} ds = 6V$$

12. (a) Evaluate using Gauss' divergence theorem $\iiint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelepiped bounded by the planes $x=0, y=0, z=0, x=1, y=2, z=3$.(b) Evaluate $\iint_S (\text{Curl } \vec{F}) \cdot \hat{n} ds$ by Stoke's theorem, where $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$.

OR

13. (a) Evaluate $\iiint_S \vec{F} \cdot \hat{n} ds$ using divergence theorem, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over rectangular box $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.(b) Evaluate by Stoke's theorem $\oint_C (\sin z dx - \cos x dy + \sin y dz)$, where C is the boundary of rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1, z = 3$.