61505

Fifth Semester B.A./B.Sc. Degree Examinations, March/April 2021

(CBCS Scheme – Freshers)

Mathematics

Paper V — MATHEMATICS – V

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer all Parts.

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) In a ring $(R, +, \cdot)$, prove that $(-a)\cdot(-b) = a\cdot b$; $\forall\ a, b \in R$.

(b) Define :

(i) Left ideal

(ii) Right ideal.

(c) Give an example of

(i) a ring without unity

(ii) an integral domain

(d) Find the unit vector normal to the surface $x^2y + 2xz = 4$.

(e) If $\phi = 2x^2 - 5y^2 + 3z^2$, find $\nabla^2\phi$.

(f) Prove that $\nabla\Delta = \delta^2$.

(g) Write the Newton's divided difference formula.

(h) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$rd rule given :

x	0	1	2	3	4	5	6
$y = f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

PART – B

Answer **TWO** full questions : **(2 × 10 = 20)**

2. (a) Prove that the intersection of any two subrings is a subring. Give an example to show that the union of two subrings of a ring need not be a subring.

(b) Prove that $(Z_5, \oplus_5, \otimes_5)$ is an integral domain with respect to addition and multiplication modulo 5.

Or

3. (a) Show that the necessary and sufficient condition for a non-empty subset S of a ring R to be a subring are

(i) $a \in S,\ b \in S \Rightarrow a - b \in S$

(ii) $a \in S,\ b \in S \Rightarrow ab \in S$.

(b) Show that the set of all matrices of the form $S = \left\{ \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \Big/ \forall\, a, b \in Z \right\}$ is a left ideal but not a right ideal in the ring of all 2×2 matrices with elements as integers.

4. (a) Prove that a ring is without zero divisors if and only if cancellation laws holds good.

(b) If $f : R \to R'$ be a homomorphism then show that $\ker f$ is an ideal of R.

Or

5. (a) Find all principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ with respect to $\oplus_8$ and $\otimes_8$.

(b) State and prove fundamental theorem of homomorphisms of rings.

PART – C

Answer **TWO** full questions : **(2 × 10 = 20)**

6. (a) Find the directional derivative of $\phi(x, y, z) = x^4 + y^4 + z^4$ at a point $P(-1, 2, 3)$ in the direction towards the point $Q(2, -1, -1)$.

(b) Show that the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ intersect orthogonally at the point $(1, -1, 2)$.

Or

7. (a) If n is a non-zero constant, then show that $\nabla^2 r^n = n(n+1)r^{n-2}$. Deduce that when $r \neq 0$, r^n is harmonic if $n = -1$.

(b) If the vector $\vec{F} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal, then find 'a'.

8. (a) If $\vec{f}$ and $\vec{g}$ are two vector fields, then prove that

$$div(\vec{f} \times \vec{g}) = \vec{g} \cdot curl\,\vec{f} - \vec{f} \cdot curl\,\vec{g}.$$

(b) If $\vec{F} = x^2 y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$, then find $curl\,(curl\,\vec{F})$.

Or

9. (a) If $\vec{a}$ is a constant vector, then find $curl\,(\vec{r} \times \vec{a})$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(b) If $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$, then find $div\,\vec{F}$ and $curl\,\vec{F}$.

PART – D

Answer **TWO** full questions : **(2 × 10 = 20)**

10. (a) Find the polynomial of degree two which takes the values :

x	0	1	2	3	4
$f(x)$	1	2	4	7	11

(b) Obtain the function whose first difference is $6x^2 + 10x + 11$.

Or

11. (a) Find the value of $f(1.4)$ from the table :

x	1	2	3	4	5
$f(x)$	10	26	58	112	194

(b) By separation of symbols, prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \cdots \infty = \frac{x}{1-x} u_1 + \frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \cdots.$$

12. (a) Using Newton's divided difference formula of interpolation find $f(6)$ from the following data :

x	3	7	9	10
$f(x)$	168	120	72	63

(b) Evaluate $\int_0^{0.6} e^{-x^2}\, dx$ by taking 6 subintervals, by using Simpson's $\frac{1}{3}$rd rule.

Or

13. (a) Find x when $y = 7$ using Lagrange's inverse interpolation formula from the data.

x	1	3	4
y	4	12	19

(b) Find the value of $\int_1^5 \log_{10} x\, dx$ taking 8 subintervals correct to four decimal places by Trapezoidal rule.

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