**61506**

# Fifth Semester B.A./B.Sc. Degree Examinations, March/April 2021

*(CBCS Scheme)*

## Paper VI – MATHEMATICS

*Time : 3 Hours]* *[Max. Marks : 70*

*Instructions to Candidates : Answer all Parts.*

PART – A

Answer any **FIVE** questions : **(5 × 2 = 10)**

1. (a) Write the Euler's equation when $f$ is independent of $x$.

   (b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} \left[y^2 + 4(y^1)^2\right] dx$.

   (c) Define geodesic on a surface.

   (d) Evaluate $\int_C 5x\,dx + y\,dy$ where $C$ is the curve $y = 2x^2$ from (0, 0) to (1, 2).

   (e) Evaluate $\int_1^2 \int_0^{x^2} x(x^2 + y^2)\,dy\,dx$.

   (f) Evaluate $\int_0^1 \int_0^1 \int_0^1 dx\,dy\,dz$.

   (g) State Stoke's theorem.

   (h) Show that the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\pi ab$ using Green's theorem.

PART – B

Answer **TWO** full questions : **(2 × 10 = 20)**

2. (a) Find the extremal of the functional $I = \int_{x_1}^{x_2} \left[y^2 + (y^1)^2 + 2ye^x\right] dx$.

   (b) Prove that the necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y^1)\,dx$ with $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0$.

Or

3. (a) Show that the extremal of $I = \int_{x_1}^{x_2} \sqrt{y\left[1 + (y^1)^2\right]}\, dx$ is a parabola.

(b) Find the curve on which the functional $I = \int_0^1 \left[(y^1)^2 + 12xy\right] dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremed.

4. (a) Show that a heavy cable hangs freely under gravity between two fixed points is a Catenary.

(b) Find the extremal of the functional $\int_0^1 \left[x + y + (y^1)^2\right] dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$.

Or

5. (a) Find the extremal of the functional $\int_0^1 (y^1)^2\, dx$ subject to the constraint $\int_0^1 y\, dx = 1$ and having $y(0) = 0$ and $y(1) = 1$.

(b) Find the geodesic on a surface of right circular cylinder.

PART – C

Answer **TWO** full questions : **(2 × 10 = 20)**

6. (a) Evaluate $\int_C (x^2 + 2y^2x)dx + (x^2y^2 - 1)dy$ around the boundary of the region defined by $y^2 = 4x$ and $x = 1$.

(b) Evaluate $\int_C (xy + z^2)dS$ where $C$ is the arc of the helix $x = \cos t$, $y = \sin t$, $z = t$ joining the points $(1, 0, 0)$ and $(-1, 0, \pi)$.

Or

7. (a) Change the order of integration and hence evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2\, dy\, dx$.

(b) Find the surface area of the portion of the cylinder $x^2 + z^2 = a^2$ which lies inside the cylinder $x^2 + y^2 = a^2$.

8. (a) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\, dx\, dy$ by changing into polar co-ordinates.

(b) Find the volume of tetrahedron formed by the planes $x=0$, $y=0$, $z=0$ and $6x+4y+3z=12$.

Or

9. (a) Evaluate $\iiint_R xyz\, dx\, dy\, dz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ by changing it to spherical polar co-ordinates.

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{a^2-x^2-y^2-z^2}}$.

PART – D

Answer **TWO** full questions : **(2 × 10 = 20)**

10. (a) Verify Green's theorem in the plane for $\oint_C xy\, dx + yx^2\, dy$ where, $C$ is the curve enclosing the region bounded by the curve $y=x^2$ and the line $y=x$.

(b) State and prove Gauss' Divergence theorem.

Or

11. (a) Using Green's theorem evaluate $\int_C e^{-x} \sin y\, dx + e^{-x} \cos y\, dy$ where, $C$ is the rectangle with vertices (0, 0), $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$.

(b) Using divergence theorem, show that

(i) $\iint_S \vec{r}.\hat{n}\, dS = 3V$ and (ii) $\iint_S \left(\nabla r^2\right).\hat{n}\, dS = 6V$.

12. (a) Using the divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\, dS$ where, $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and $S$ is the surface of the sphere $x^2+y^2+z^2=a^2$.

(b) Evaluate by Stoke's theorem $\oint_C yz\, dx + zx\, dy + xy\, dz$ where, $C$ is the curve $x^2+y^2=1$, $z=y^2$.

Or

13. (a) Evaluate $\iint_S \vec{F}.\hat{n}\,dS$ using divergence theorem where, $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over rectangular box $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$.

(b) Evaluate by Stoke's theorem $\oint_C (\sin z\, dx - \cos x\, dy + \sin y\, dz)$ where, $C$ is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and $z = 3$.

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