61323

Third Semester B.A./B.Sc. Degree Examinations, August/September 2021

(CBCS Scheme)

MATHEMATICS

Paper III

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer all Parts.

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) Write the order of elements of the group $z_4 = \{0, 1, 2, 3\}$ with respect to the addition modulo 4.

(b) Define Cyclic group.

(c) Show that $\left\{\frac{1}{n}\right\}$ is a monotonically decreasing sequence.

(d) Examine the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.

(e) State Raabe's test for the series of positive terms.

(f) Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$.

(g) Prove that every differentiable function is a continuous function.

(h) Evaluate $\lim_{x \to 0}\left(\frac{x - \sin x}{x^2}\right)$.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) Prove that in a group G, $O(a) = O(a^{-1}), \forall a \in G$.

(b) Find all right and left cosets of the subgroup $H = \{0, 2, 4\}$ of the group $(z_6, \oplus_6)$.

(c) State and prove Lagranges theorem in groups.

Or

3. (a) Prove that there is one-to-one correspondence between the set of all right cosets and the set of all left cosets of a subgroup H of a group G.

(b) If 'a' and 'b' are any two arbitrary elements of G, then show that $O(a) = O(b^{-1}ab)$.

(c) If 'a' is a generator of a cyclic group of order 10, how many generators are there? What are they?

PART – C

Answer **TWO** full questions : **(2 × 15 = 30)**

4. (a) Prove that every convergent sequence is bounded.

(b) Show that the sequence $\{a_n\}$, where $a_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is convergent.

(c) Examine the behaviour of the sequences

(i) $\left\{\sqrt{n^2+1} - 1\right\}$

(ii) $\left\{\frac{1+(-1)^n n}{n+1}\right\}$

Or

5. (a) Prove that monotonically decreasing sequence which is bounded below is convergent.

(b) Discuss the nature of the sequence $\left\{n^{1/n}\right\}$.

(c) Test the convergence of the sequence

(i) $\{n(\log(n+1) - \log n)\}$

(ii) $\left\{\frac{n+1}{n}\right\}$

6. (a) State and prove D'Alembert's Ratio test for the convergence of positive terms.

(b) Discuss the convergence of the series :

$$\frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \cdots$$

(c) Sum to infinity the series $1 + \frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \cdots$

Or

7. (a) State and prove Cauchy's root test for a series of positive terms.

(b) Examine the convergence of the series $\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+\cdots$

(c) Sum to infinity the series $1-\frac{3}{5}+\frac{3\cdot 5}{5\cdot 10}-\frac{3\cdot 5\cdot 7}{5\cdot 10\cdot 15}+\cdots$

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Examine the continuity of $f(x)=\begin{cases}1-3x, & \text{for } x\le 1\\ x^2-5x+2, & \text{for } x>1\end{cases}$ at $x=1$.

(b) State and prove Lagrange's mean value theorem.

(c) Evaluate $\lim_{x\to 0}(1+\sin x)^{\cot x}$.

Or

9. (a) Examine the differentiability of $f(x)=\begin{cases}x^2-1, & \text{for } x\ge 1\\ 1-x, & \text{for } x<1\end{cases}$ at $x=1$.

(b) State and prove Cauchy's Mean Value Theorem.

(c) Expand $\log(1+\sin x)$ upto term containing x^3 using Maclaurin's expansion.