First Semester B.A./B.Sc. Degree Examination, August/September 2021

(CBCS Scheme)

Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer ALL Parts.

PART – A

Answer any **FIVE** questions : **(5 × 2 = 10)**

1. (a) Find the value of λ in order that the matrix $A=\begin{bmatrix}6 & \lambda & -1\\ 2 & 3 & 1\\ 3 & 4 & 2\end{bmatrix}$ has rank 2.

(b) Find the Eigen values of the matrix $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$.

(c) Find the nth derivative of $\frac{1}{(5x-2)^3}$.

(d) If $z=x^2+y^2-3xy$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

(e) Evaluate $\int_0^{\frac{\pi}{2}}\cos^8 x\,dx$.

(f) Evaluate $\int_0^{\frac{\pi}{2}}\sin^6 x\cos^4 x\,dx$.

(g) Find the angle between the line $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z+4}{3}$ and the plane $2x+3y-z=4$.

(h) If the two spheres $x^2+y^2+z^2+6z-k=0$ and $x^2+y^2+z^2+10y-4z-8=0$ cut orthogonally find k.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & -2 \\ -2 & -5 & 1 & 2 \\ -3 & -8 & 5 & 2 \\ -5 & -12 & -1 & 6 \end{bmatrix}$.

(b) For what values of λ and μ the equations $x+2y+3z=5$, $x+3y-z=4$, $x+4y+\lambda z=\mu$ have

(i) no solution,

(ii) a unique solution

(iii) an infinite number of solution.

(c) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

Or

3. (a) Reduce the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ to the normal form and hence find the rank.

(b) Show that the equations $4x-5y+z=2$, $3x+y-2z=9$, $x+4y+z=5$ are consistent and solve them.

(c) If $A=\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ find A^3 by using Cayley-Hamilton theorem.

PART – C

Answer **TWO** full questions : **(2 × 15 = 30)**

4 (a) Find the nth derivative of $\dfrac{x}{1+3x+2x^2}$.

(b) Find the nth derivative of $e^x \sin x \cos 2x$.

(c) If $y=\tan^{-1}x$ prove that $\left(1+x^2\right)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$.

Or

5. (a) If $u=f(y+ax)+g(y-ax)$, show that $\dfrac{\partial^2 u}{\partial x^2}=a^2\dfrac{\partial^2 u}{\partial y^2}$.

(b) State and prove Euler's theorem for homogeneous function of x and y.

(c) If $f=f(u,v,w)$ and $u=\dfrac{x}{y}$, $v=\dfrac{y}{z}$, $w=\dfrac{z}{x}$, show that $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}+z\dfrac{\partial f}{\partial z}=0$.

6. (a) Find $\frac{dz}{dt}$ if $z=x^2+y^2$ where $x=e^t \cos t$, $y=e^t \sin t$.

(b) If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}=r^2\sin^2\theta$.

(c) Obtain the reduction formula for $\int \sin^n x\, dx$ where n is a positive integer.

Or

7. (a) Evaluate $\int_0^4 x^3\sqrt{4x-x^2}\,dx$.

(b) Evaluate $\int_0^{\pi} x\cos^4 x\, dx$.

(c) Using Leibnitz's rule of differentiation under integral sign evaluate $\int_0^1 \frac{x^\alpha-1}{\log x}dx$ where $\alpha>0$ is a parameter.

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Find the equation of the plane coaxial with the planes $x-2y+Z-7=0$ and $2x+3y-4z=0$ and cutting intercept 4 units on the x - axis.

(b) Show that the lines $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{1}$ and $\frac{x-2}{3}=\frac{y-2}{2}=\frac{z-6}{4}$ are coplanar and find the equation of the plane containing them.

(c) Find the equation of the sphere which passing through the origin and the points $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.

Or

9. (a) Find the shortest distance between the lines $\frac{x-2}{3}=\frac{y-6}{-2}=\frac{z-5}{-2}$ and $\frac{x-5}{2}=\frac{y-3}{1}=\frac{z+4}{-6}$.

(b) Find the equation of the right circular cone whose axis is $\frac{x-1}{-1}=\frac{y-2}{3}=\frac{z-3}{3}$ and a generator is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

(c) Find the equation of the right circular cylinder of radius 3 and the axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.