61625

Sixth Semester B.A./B.Sc. Degree Examinations, September/October 2021

(CBCS – Freshers – Semester Scheme)

Paper VII – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer ALL Parts.

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

(a) In a vector space V over F, show that $a \cdot 0 = 0, \ \forall \ a \in F, \ 0 \in V$.

(b) Show that $w = \{(a, 0, 0) \mid a \in R\}$ is a subspace of $V_3(R)$.

(c) Show that the vectors $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$ and $\alpha_3 = (0, 0, 1)$ are linearly independent.

(d) Verify $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x + y, y)$ is a linear transformation.

(e) Define orthogonal curvilinear coordinate system.

(f) Verify the condition for integrability

$z^2 \ dx + (z^2 - 2yz) \ dy + (2y^2 - yz - xz) \ dz = 0$.

(g) Form the partial differential equation by eliminating arbitrary constants for the equation $z = (x + a)(y + b)$.

(h) Solve $p^2 + q^2 = 3$.

PART – B

Answer **TWO** full questions : **(2 × 10 = 20)**

2. (a) Show that $V = \{(x_1, x_2, x_3) \mid x_1, x_2, x_3 \in R\}$ is a vector space over R.

(b) Verify $W = \{(x, y, z) \mid 2x + 3y + z = 0\}$ is a subspace of $V_3(R)$.

Or

3. (a) Prove that a set of non-zero vectors $\{\alpha_1, \alpha_2, \alpha_3, \dots \alpha_n\}$ of a vector space $V(F)$ is linearly dependent if and only if one of the vectors say α_k $(2 \le k \le n)$ is expressed as the linear combination of its proceeding vectors.

(b) Find the basis and dimension of the vector space spanned by the vectors (2, –3, 1), (3, 0, 1), (0, 2, 1) and (1, 1, 1).

4. (a) Find the linear transformation $T : R^3 \to R^3$ such that $T(1, 0, 0) = (4, 5, 8)$, $T(1, -1, 0) = (8, 10, 8)$, $T(0, 1, 1) = (-3, -4, -7)$.

(b) Find the matrix of linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x - y, x, 3x + y)$ w.r.t. standard bases.

Or

5. (a) State and prove Rank-Nullity theorem.

(b) Show that the linear transformation $T : V_3(R) \to V_3(R)$ defined by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ is non-singular, where $\{e_1, e_2, e_3\}$ is the standard basis of $V_3(R)$.

PART – C

Answer **TWO** full questions : **(2 × 10 = 20)**

6. (a) Verify the condition of integrability and solve

$$3x^2\ dx + 3y^2\ dy - (x^3 + y^3 + e^{2z})\ dz = 0.$$

(b) Solve $p \cot x + q \cot y = \cot z$.

Or

7. (a) Show that cylindrical coordinate system is orthogonal curvilinear coordinate system.

(b) Express $\vec{f} = zi - 2xj + yk$ in spherical polar coordinates and find f_r, f_θ, f_ϕ.

8. (a) Solve $\dfrac{dx}{mz - ny} = \dfrac{dy}{nx - lz} = \dfrac{dz}{ly - mx}$.

(b) Solve $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$.

Or

9. (a) Express $\vec{f} = 3yi + x^2 j - z^2 k$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z.

(b) Express $\vec{f} = 3yi + 2zj + zk$ in spherical polar coordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **TWO** full questions : **(2 × 10 = 20)**

10. (a) Form the partial differential equation by eliminating arbitrary function from $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

(b) Solve $p(1+q^2) = q(z-a)$.

Or

11. (a) Solve $(D^2 - DD' - 2D'^2)Z = e^{x+2y}$.

(b) Solve $p + q = \sin x + \sin y$.

12. (a) Solve by Charpit's method : $Z = pq$.

(b) Solve $(D^2 - 2DD' + D'^2)Z = 12xy$.

Or

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from the rest from this position, find the displacement $y(x, t)$.

(b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the condition

(i) $u(0, t) = 0$, $u(1, t) = 0$ for all t.

(ii) $u(x, 0) = x^2 - x$, $0 \le x \le 1$.