3. a) Prove that a ring R is without zero divisors if and only if cancellation laws hold in R.

b) Find all the principal ideals of the ring

$R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. $+_6$ and x_6.

4. a) If $f : R \to R'$ be a homomorphism of R into R′ then show that Ker f is an ideal of R.

b) State and prove fundamental theorem of homomorphism of rings.

OR

5. a) Show that $f : R_1 \to R$ defined by $f\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = a, \forall \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \in R$ is an isomorphism where $R_1 = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \Big/ a \in R \right\}$.

b) Prove that an ideal S of the ring of integers (z, +, .) is maximal if and only if S is generated by some prime integer.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz - 9x = 0$ intersect orthogonally at the point (1, –1, 2).

b) If $\vec{F} = \text{grad}\,(2x^3y^2z^4)$ find $\text{div}\left(\vec{F}\right)$ and $\text{curl}\left(\vec{F}\right)$.

OR

7. a) Prove that $\nabla^2(r^n) = n\,(n + 1)r^{n-2}$ where n is a non-zero constant. Also show that r^n is harmonic if $n = -1$.

b) If the vector $\vec{F} = (3x + 3y + 4z)\,\hat{i} + (x - ay + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$ is Solenoidal find ‘a’.