



3. a) Find the function y which makes the integral $\int_{x_1}^{x_2} [y^2 + 4(y')^2] dx = 0$ an extremum.

b) Find the geodesics on a surface given that the arc length on the surface is

$$S = \int_{x_1}^{x_2} \sqrt{x[1+(y')^2]} dx.$$

4. a) If a cable hangs freely under gravity from two fixed points, then show that the shape of the curve is a catenary.

b) Find the extremal of the functional $\int_0^1 [(y')^2 + x^2] dx$ subject to the constraint $\int_0^1 y dx = 2$ and having end conditions $y(0) = 0, y(1) = 1$.

OR

5. a) Find the extremal of the functional $I = \int_0^{\pi} [(y')^2 - y^2] dx$ under the conditions

$$y(0) = 0, y(\pi) = 1 \text{ subject to the condition } \int_0^{\pi} y dx = 1.$$

b) Find the extremal of the functional $I = \int_{x_1}^{x_2} [1 + x y' + x(y')^2] dx$.

PART - C

Answer **two full** questions :

(2×10=20)

6. a) Evaluate $\int_C [(2x + y)dx + (3y + x) dy]$ along the line joining (0,1) and (2, 5)

b) Evaluate $\iint_R xy dx dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

OR

7. a) Change the order of integration and evaluate $\int_0^3 \int_0^{\sqrt{4-y}} (x+y) dx dy$.

b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ by double integration.