8. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.

b) Evaluate $\iint_R xy\,(x^2+y^2)^{3/2}\,dxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$ by transforming to polar co-ordinates.

OR

9. a) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=3$ and $z=0$.

b) Evaluate $\iiint_R z\,(x^2+y^2)\,dx\,dy\,dz$, $x^2+y^2 \le 1$; $2 \le z \le 3$ by changing to cylindrical polar co-ordinates.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) State and prove Green's theorem in a plane.

b) By using Green's theorem, evaluate $\int_C (y-\sin x)\,dx + \cos x\,dy$, where C is the triangle in the xy-plane bounded by the lines $y=0$, $x=\frac{\pi}{2}$ and $y=\frac{2x}{\pi}$.

OR

11. a) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ using divergence theorem, where $\vec{F}=(x^2-yz)\,\hat{i}+(y^2-zx)\,\hat{j}+(z^2-xy)\,\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

b) Evaluate $\int_C e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$, using Green's theorem, where C is the rectangle with vertices (0, 0), (π, 0), $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$.

12. a) Using Stoke's theorem evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}=y^2\,\hat{i}+x^2\,\hat{j}-(x+z)\,\hat{k}$ and C is the boundary of the triangle with vertices (0, 0, 0) , (1, 0, 0) and (1, 1, 0).