DCMT 201

II Semester B.Sc. Examination, October/November 2022
(NEP Scheme)
MATHEMATICS – II
Algebra – II and Calculus – II

Time : 2½ Hours Max. Marks : 60

***Instruction** : Answer **all** the Parts.*

PART – A

I. Answer **any six** of the following : **(6×2=12)**

1) Let G be a group and $*$ is defined by $a * b = \frac{ab}{7}$, prove that G is an abelian.

2) Define order of an element.

3) Verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$, given $u = x^2 + y^2$.

4) If $u = 3x + 5y$, $v = 4x - 3y$, show that $\frac{\partial(u,v)}{\partial(x,y)} = -29$.

5) Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x \, dx$.

6) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x . \cos^3 x \, dx$.

7) Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)\,dx + (2y - x)\,dy$ along the curve $y = x^2$.

8) Evaluate $\int_1^2 \int_0^3 y \, dx \, dy$.

PART – B

II. Answer **any three** of the following : **(3×4=12)**

1) Show that $G = \{2, 4, 6, 8\}$ is an abelian group $(Z_{10}, \otimes_{10})$.

2) Find the number of generators of cyclic groups of order (i) 6 (ii) 8.

P.T.O.

3) State and prove Lagrange's theorem.

4) Let G be a group for any three subsets H, K, L. Prove that (HK)L = H(KL) when $HK = \{hk | h \in H, k \in K\}$.

5) Find the right cosets of the subgroup H = {0, 3} in the group $(Z_6, +_6)$.

PART – C

III. Answer **any three** of the following : **(3×4=12)**

1) If $u = xy + yz + zx$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2(x + y + z)$.

2) State and prove Euler's theorem for homogenous functions of x and y.

3) Find $\frac{du}{dt}$, if $u = e^x \sin y$, where $x = \log t$, $y = t^2$.

4) By using Maclaurin's expansion, expand $y = e^x \cos y$ upto second degree.

5) If $u = x + y + z$, $v = y + z$, $w = z$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 1$.

PART – D

IV. Answer **any three** of the following : **(3×4=12)**

1) Obtain the reduction formula for $\int \sin^n x \, dx$ where n is the positive integer.

2) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^6 x \, dx$.

3) Evaluate $\int_0^{\pi} x \sin^7 x \, dx$.

4) Show that $\int_0^{\pi} x \sin^4 x \cos^2 x \, dx = \frac{\pi^2}{32}$.

5) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2} \, dx$.

PART – E

V. Answer **any three** of the following : **(3×4=12)**

1) Evaluate $\int_C xy\,dx$ around the circle $x^2 + y^2 = 1$.

2) Evaluate $\int_0^2 \int_1^2 (x^2 + y^2)\,dx\,dy$.

3) Evaluate $\int_0^1 \int_{x^2}^{x} (x^2 + 3y + 2)\,dy\,dx$.

4) Evaluate $\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2)\,dx\,dy\,dz$.

5) Prove that $\int_0^3 \int_0^2 \int_0^1 xyz\,dx\,dy\,dz = \frac{9}{2}$.