

II Semester B.Sc. Examination, October/November 2022

(NEP Scheme)

MATHEMATICS - II

Algebra - II and Calculus - II

Max. Marks : 60

Time : 2½ Hours

Instruction : Answer all the Parts.

PART - A

(6×2=12)

I. Answer any six of the following :

- 1) Let G be a group and $*$ is defined by $a * b = \frac{ab}{7}$, prove that G is an abelian.
 2) Define order of an element.

3) Verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$, given $u = x^2 + y^2$.

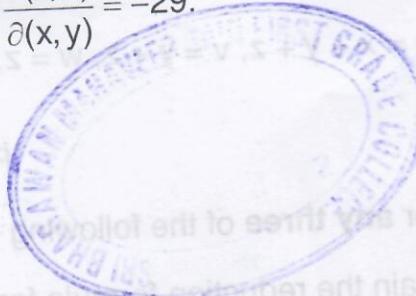
4) If $u = 3x + 5y$, $v = 4x - 3y$, show that $\frac{\partial(u, v)}{\partial(x, y)} = -29$.

5) Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$.

6) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cdot \cos^3 x dx$.

7) Evaluate $\int_{(0,1)}^{(2,5)} (3x+y) dx + (2y-x) dy$ along the curve $y = x^2$.

8) Evaluate $\iint_{10}^{23} y dx dy$.



PART - B

(3×4=12)

II. Answer any three of the following :

- 1) Show that $G = \{2, 4, 6, 8\}$ is an abelian group $(\mathbb{Z}_{10}, \otimes_{10})$.
 2) Find the number of generators of cyclic groups of order (i) 6 (ii) 8.

- 3) State and prove Lagrange's theorem.
- 4) Let G be a group for any three subsets H, K, L . Prove that $(HK)L = H(KL)$
when $HK = \{hk | h \in H, k \in K\}$.
- 5) Find the right cosets of the subgroup $H = \{0, 3\}$ in the group $(\mathbb{Z}_6, +_6)$.

PART - C

III. Answer **any three** of the following :

(3x4=12)

- 1) If $u = xy + yz + zx$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2(x + y + z)$.
- 2) State and prove Euler's theorem for homogenous functions of x and y .
- 3) Find $\frac{du}{dt}$, if $u = e^x \sin y$, where $x = \log t$, $y = t^2$.
- 4) By using Maclaurin's expansion, expand $y = e^x \cos y$ upto second degree.
- 5) If $u = x + y + z$, $v = y + z$, $w = z$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 1$.

PART - D

IV. Answer **any three** of the following :

(3x4=12)

- 1) Obtain the reduction formula for $\int \sin^n x dx$ where n is the positive integer.
- 2) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^6 x dx$.
- 3) Evaluate $\int_0^{\pi} x \sin^7 x dx$.
- 4) Show that $\int_0^{\pi} x \sin^4 x \cos^2 x dx = \frac{\pi^2}{32}$.
- 5) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2} dx$.



PART - E

V. Answer any three of the following :

(3x4=12)

1) Evaluate $\int_C xy \, dx$ around the circle $x^2 + y^2 = 1$.

2) Evaluate $\iint_{0,1}^{2,2} (x^2 + y^2) \, dx \, dy$.

3) Evaluate $\iint_{0, x^2}^{1, x} (x^2 + 3y + 2) \, dy \, dx$.

4) Evaluate $\iiint_{0,0,0}^{a,a,a} (x^2 + y^2 + z^2) \, dx \, dy \, dz$.

5) Prove that $\iiint_{0,0,0}^{3,2,1} xyz \, dx \, dy \, dz = \frac{9}{2}$.