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VI Semester B.A./B.Sc. Examination, September/October 2022
(CBCS) (F+R) (2016-17 and Onwards)
MATHEMATICS
Mathematics – VIII

Time : 3 Hours Max. Marks : 70

Instruction : *Answer* ***all*** *Parts.*

PART – A

I. Answer **any five** questions. **(5×2=10)**

1) a) Evaluate $\lim_{z \to -i} \frac{z^2+1}{z^6+1}$.

b) Show that $\left|\frac{z-2}{z+2}\right| = 3$ represents a circle.

c) Show that $u = e^x \cos y + xy$ is harmonic.

d) Define Bilinear transformation.

e) Show that $f(z) = \sin z$ is analytic.

f) State Liouville's theorem.

g) Find the real root of the equation $x^3 - x - 2 = 0$ over the interval (1.5, 2) upto two approximation by Bisection method.

h) Write iteration formula for Runge-Kutta method of fourth order.

PART – B

II. Answer **four full** questions. **(4×10=40)**

2) a) Find the locus of the point z satisfying the relation $|z + 1| + |z - 1| = 4$.

b) Prove that the necessary condition for a function $f(z) = u+iv$ to be analytic is $u_x = v_y$ and $u_y = -v_x$.

OR

P.T.O.

3) a) Prove that $\lim_{z \to 0} \left(\frac{\bar{z}}{z}\right)$ does not exists.

b) Show that f(z) = log z is analytic and hence prove that f'(z) = $1/z$.

4) a) Find the analytic function whose real part is $x^2 - y^2 + \frac{x}{x^2 + y^2}$.

b) Find the orthogonal trajectory of the family of curves $x^2 - y^2 + x = c$.

OR

5) a) Show that an analytic function with constant modulus is a constant.

b) Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.

6) a) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ along the line $y = \frac{x}{2}$.

b) State and prove Cauchy's Integral Formula.

OR

7) a) Evaluate $\oint_C \frac{1}{z(z-1)} dz$ where 'C' is the circle $|z| = 3$.

b) State and prove fundamental theorem of algebra.

8) a) Prove that the Bilinear transformation preserves the cross ratio of four points.

b) Discuss the transformation $W = Z^2$.

OR

9) a) Show that the transformation $W = 1/z$ transforms circle into circle or to a straight line.

b) Find the Bilinear transformation which maps Z = 1, i, –1 onto W = 0, 1, ∞.

PART – C

III. Answer **two full** questions. **(2×10=20)**

10) a) Find the root of the equation $x^3 - 4x + 1 = 0$ by Regula – Falsi method upto three decimal places.

b) Find the cube root of 24, correct to three decimal places by Newton-Raphson method.

OR

11) a) Solve by Gauss – Jacobi method :

$x + y + 54z = 110$

$27x + 6y - z = 85$

$6x + 15y + 2z = 72.$

b) Find the largest eigenvalue of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ by power method.

12) a) Use Taylor's series method to find y(0.1) considering terms upto the third degree given $\frac{dy}{dx} = 1 + xy$ and $y(0) = 1$.

b) Using Euler's method solve $\frac{dy}{dx} = x - y$ for $x = 0\ (0.1)\ 0.5$ given $y = 1$ when $x = 0$.

OR

13) a) Using Euler's modified method find y (0.1), given $\frac{dy}{dx} = x^2 + 1$, $y(0) = 1$.

b) Using Runge-Kutta method, find y(0. 2) for $\frac{dy}{dx} = x + y$; $y(0) = 1$ taking $h = 0.2$.
