

## VI Semester B.A./B.Sc. Examination, September/October 2022 (CBCS) (F+R) (2016-17 and Onwards) MATHEMATICS Mathematics – VIII

Time: 3 Hours Max. Marks: 70

Instruction: Answer all Parts.

PART - A MI OMISMS HIS HER WORLD IS (2)

I. Answer any five questions.

 $(5\times2=10)$ 

- 1) a) Evaluate  $\lim_{z \to -i} \frac{z^2 + 1}{z^6 + 1}$ .
  - b) Show that  $\left| \frac{z-2}{z+2} \right| = 3$  represents a circle.
  - c) Show that  $u = e^x \cos y + xy$  is harmonic.
  - d) Define Bilinear transformation.
  - e) Show that  $f(z) = \sin z$  is analytic.
  - f) State Liouville's theorem.
  - g) Find the real root of the equation  $x^3 x 2 = 0$  over the interval (1.5, 2) upto two approximation by Bisection method.
  - h) Write iteration formula for Runge-Kutta method of fourth order.

PART - B

II. Answer four full questions.

 $(4 \times 10 = 40)$ 

- 2) a) Find the locus of the point z satisfying the relation |z + 1| + |z 1| = 4.
- b) Prove that the necessary condition for a function f(z) = u+iv to be analytic is  $u_x = v_y$  and  $u_y = -v_x$ .

OR



- 3) a) Prove that  $\lim_{z \to 0} \left(\frac{\overline{z}}{z}\right)$  does not exists.
  - b) Show that  $f(z) = \log z$  is analytic and hence prove that  $f'(z) = \frac{1}{Z}$ .
- 4) a) Find the analytic function whose real part is  $x^2 y^2 + \frac{x}{x^2 + y^2}$ .
  - b) Find the orthogonal trajectory of the family of curves  $x^2 y^2 + x = c$ .

    OR
- 5) a) Show that an analytic function with constant modulus is a constant.
  - b) Show that  $u = e^x \sin y + x^2 y^2$  is harmonic and find its harmonic conjugate.
- 6) a) Evaluate  $\int_0^{2+i} (\bar{z})^2 dz$  along the line  $y = \frac{x}{2}$ 
  - b) State and prove Cauchy's Integral Formula.

OR

- 7) a) Evaluate  $\oint_C \frac{1}{z(z-1)} dz$  where 'C' is the circle |z| = 3.
  - b) State and prove fundamental theorem of algebra.
- 8) a) Prove that the Bilinear transformation preserves the cross ratio of four points.
  - b) Discuss the transformation  $W = Z^2$ .

OR

- 9) a) Show that the transformation  $W = \frac{1}{Z}$  transforms circle into circle or to a straight line.
- b) Find the Bilinear transformation which maps Z=1, i, -1 onto  $W=0, 1, \infty$ .

2) a) Find the locus of the poin  $2 - TRAP_{10}$  the relation |z + 1| + |z - 1| = 4.

III. Answer two full questions.

(2×10=20)

- 10) a) Find the root of the equation  $x^3 4x + 1 = 0$  by Regula Falsi method upto three decimal places.
  - Find the cube root of 24, correct to three decimal places by Newton-Raphson method.



11) a) Solve by Gauss - Jacobi method:

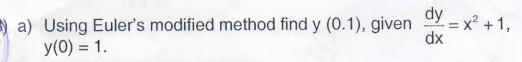
$$x + y + 54 z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$
.

- b) Find the largest eigenvalue of the matrix  $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$  by power method.
- 12) a) Use Taylor's series method to find y(0.1) considering terms upto the third degree given  $\frac{dy}{dx} = 1 + xy$  and y(0) = 1.
  - b) Using Euler's method solve  $\frac{dy}{dx} = x y$  for x = 0 (0.1) 0.5 given y = 1 when x = 0.

OR



b) Using Runge-Kutta method, find y(0. 2) for  $\frac{dy}{dx} = x + y$ ; y (0) = 1 taking h = 0.2.

