IV Semester B.A./B.Sc. Examination, September/October 2022
(CBCS) (F + R)
MATHEMATICS – IV

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

   a) Show that every quotient group of an abelian group is abelian.

   b) If $f : G \to G'$ be an isomorphism then prove that $f(e) = e'$ where e and e′ are identities of G and G′ respectively.

   c) Find a_0 in the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.

   d) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at (1, 1).

   e) Find the Laplace transform of sin5t cos2t.

   f) Find the inverse Laplace transform of $\left[\frac{1}{(s-4)^3}\right]$.

   g) Solve : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.

   h) Show that the equation $(1 - x^2)\, y'' - 3xy' - y = 0$ is exact.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} \in H, \forall\, g, h \in G$.

   b) If $f : G \to G'$ be a homomorphism from the group G into G′ with Kernel K then prove that K is a normal subgroup of G.

   c) If $f : (z_8, +_8) \to (z_2, +_2)$ is given by $f(x) = r$ where r is the remainder when x is divided by 2. Show that f is a homomorphism.

OR

3. a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

b) If S = {1, 2, 3, 4}, $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ then find $f_o g$ and $g_o f$.

c) State and prove fundamental theorem of homomorphism.

PART – C

Answer **any two full** questions. **(2×15=30)**

4. a) Find the Fourier series of f(x) = x, in $-\pi < x < \pi$.

b) Find the half range cosine series for the function $f(x) = (x - 1)^2$ in the interval $0 < x < 1$.

c) Obtain Taylor's expansion of $e^x \cos y$ about the point $\left(1, \frac{\pi}{4}\right)$ upto second degree terms.

OR

5. a) Find the extreme values of f(x, y) = xy (a – x – y) at the point $\left(a/3, a/3\right)$.

b) Show that a rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

c) Obtain the Fourier series of the function f(x) = |x| in $(-\pi, \pi)$.

6. a) Find L $[e^{-t} \sin 4t + t \cos 2t]$.

b) Find $L^{-1}\left[\frac{s+2}{s^2 - 2s + 5}\right]$.

c) Express $f(t) = \begin{cases} 2t, & 0 < t < \pi \\ 1, & t > \pi \end{cases}$ in terms of unit step function and find L{f(t)}.

OR

7. a) Find $L\left[\frac{2s+3}{(s-1)(s+2)^2}\right]$.

b) By using the convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$.

c) Find $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$.

PART – D

Answer **one full** question. **(1×15=15)**

8\. a) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \cos 3x$.

b) $x^2y'' + xy' - y = 2x^2$ $(x > 0)$ given that $\frac{1}{x}$ is a part of its complementary function.

c) Solve : $(D^2 + 4D + 4)\, y = e^{2x} - e^{-2x}$.

OR

9\. a) Solve : $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \sin(\log x)$.

b) Solve : $\frac{dx}{dt} = 7x - y,\ \frac{dy}{dt} = 2x + 5y$.

c) Solve by the method of variation of parameters $y'' + a^2y = \sec ax$.
